Negative energies in quantum gravity

Gerard 't Hooft

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Presented at QGRAV2021
Workshop: Quantum Gravity, Higher Derivatives & Nonlocality
Zoom webinar

March 8, 2021

Negative energies *and metrics* in quantum gravity

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QFT Feynman rules are derived from a Lagrangian

$$\mathcal{L}(x) = -\tfrac{1}{2}(\partial\phi^2 + M^2\phi^2) - \tfrac{\lambda}{3!}\phi^3 + \cdots$$

and if you have indices $\mu,\ \nu,\ \cdots,$ just make sure that a gauge can be chosen such that all signs stay like this

(Time-like components of vector fields must be set to vanish, etc.)

Inverse of quadratic term, $k^2 + M^2$ is the propagator, but we need

$i\varepsilon$ prescription for the pole: $\dfrac{1}{k^2 + M^2 - i\varepsilon}$.

Using $\int_{-\infty}^{\infty} \mathrm{d}x \theta(x) e^{ikx-\varepsilon x} = \frac{1}{\varepsilon - ik}$,

The Feynman propagator: $\Delta^F(x) = -i \int \mathrm{d}^4 k \frac{e^{ik\cdot x}}{k^2 + M^2 - i\varepsilon}$;

On shell: $\Delta^{\pm}(x) = 2\pi \int \mathrm{d}^4 k \, e^{ik\cdot x} \, \delta(k^2 + M^2) \theta(\pm k^0)$.

By contour integration: $\Delta^F(x) = \theta(x^0)\Delta^+(x) + \theta(-x^0)\Delta^-(x)$;

$$\Delta^+(x) = \theta(x^0)\Delta^F(x) + \theta(-x^0)\Delta^F(x)^*.$$

Because of these identities,

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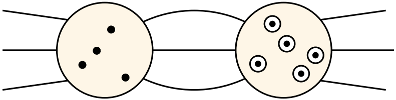
Because of these identities, *If all signs are taken correctly,* scattering matrix calculated with the Feynman rules is unitary:

$\sum$  $= 0.$

$\sum_n S|n\rangle\langle n|S^\dagger \;=\; S$ on shell $S^\dagger$ $= \mathbb{I}\,.$

propagator:

$$\frac{1}{k^2+M^2-i\varepsilon} \quad \to \quad \frac{-1}{k^2+M^2-i\varepsilon} \qquad \text{neg. metric}$$

$$\to \quad \frac{1}{k^2+M^2+i\varepsilon} \qquad \text{neg. energy.}$$

$$\to \quad \frac{1}{k^2-M^2\pm i\varepsilon} \qquad \text{negative mass}^2: \text{ tachyons, unstable}$$

On the occurrence of **negative metric** in gravity:

In gravity, we have the non renormalizable Einstein-Hilbert action

$$\mathcal{L}_{\rm tot} = \sqrt{-g}\left(\frac{1}{16\pi G_N}\,(R-2\Lambda)+\mathcal{L}^{\rm matter}\right)\ .$$

Introduce Weyl tensor $C_{\mu\nu\alpha\beta}$ defined by

$$\begin{aligned} C_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta}-\tfrac{1}{2}\left(R_{\mu\alpha}g_{\nu\beta}-(\mu\leftrightarrow\nu)-(\alpha\leftrightarrow\beta)+\left({}^{\mu}_{\nu}\leftrightarrow{}^{\alpha}_{\beta}\right)\right) \\ &+ \tfrac{1}{6}\,R\left(g_{\mu\alpha}g_{\nu\beta}-(\mu\leftrightarrow\nu)\right)\ , \end{aligned}$$

such that, in 4 dim, it is traceless $\left(g^{\mu\alpha}C_{\mu\nu\alpha\beta}=\cdots=0\right)$. One derives:

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such that, in 4 dim, it is traceless $(g^{\mu\alpha}C_{\mu\nu\alpha\beta} = \cdots = 0)$. One derives:

$$CC \equiv C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 2R_{\mu\nu}R^{\mu\nu} + \tfrac{1}{3}R^2$$

while

$$G \equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

is a total derivative and therefore topologically invariant $\to$

$$CC \equiv 2R_{\mu\nu}R^{\mu\nu} - \tfrac{2}{3}R^2$$

Using conformal invariance we can argue that this would be the only term needed to renormalize gravity. **BUT ...**

What happens if $\mathcal{L}_{\rm EH} \to \frac{\sqrt{-g}}{16\pi G_N}(R - \lambda CC)$?

Propagator tends to $1/k^4$ for large k^2:

$$\frac{1}{(k^2 - i\varepsilon)(1 + \frac{\lambda}{M^2_{\rm Planck}} k^2 - i\varepsilon)} \to \frac{1}{k^2 - i\varepsilon} - \frac{1}{k^2 + M^2_{\rm Planck}/\lambda - i\varepsilon} \ .$$

We see an extra particle with spin 2 and mass $M_{\rm Planck}/\sqrt{\lambda}$,
with wrong sign !

Actually interesting:

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We see an extra particle with spin 2 and mass $M_{\rm Planck}/\sqrt{\lambda}$,
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Actually interesting: negative metric particle means that this is a massive object that has to be *removed* from the spectrum of allowed states!

Or,

Gravity has fewer particle- or matter- states than suggested by $\mathcal{L}_{\rm EH}$.

Negative energy:

should not be accepted in any theory unless its origin is fully explained.
In BEH models, we have

$$V(\phi) = \tfrac{1}{2}\lambda(\phi^2 - F^2)^2 \quad = \quad \tfrac{1}{2}\lambda F^2 - \lambda F^2\phi^2 + \tfrac{1}{2}\lambda\phi^4 \ .$$

wrong ↑
sign

$\phi = F + \tilde{\phi}$ gives correct sign:

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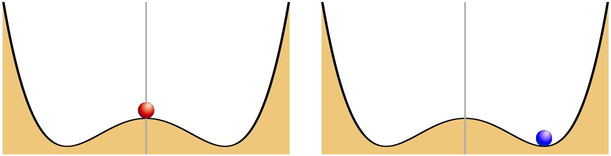
wrong ↑ sign

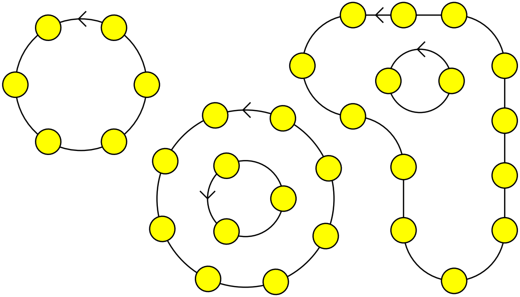
$$\phi = F+\tilde\phi \ \text{ gives correct sign:} \quad = \quad \lambda F^2\tilde\phi^2 + 2\lambda F\tilde\phi^3 + \tfrac{1}{2}\lambda\tilde\phi^4$$

↑ correct sign



Negative energies are important for understanding QM.
But first:

The generic *deterministic* model that can be handled by quantum mechanical techniques:



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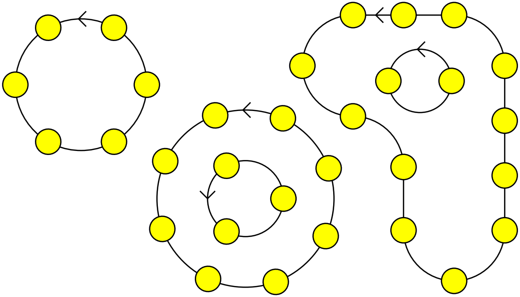

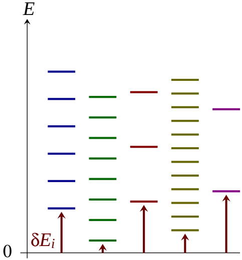

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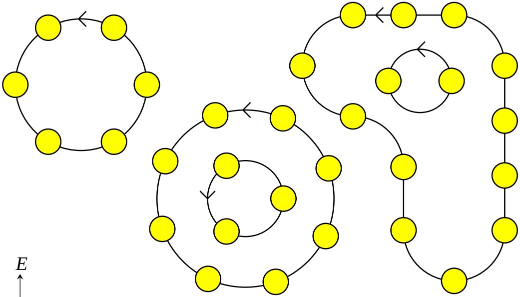

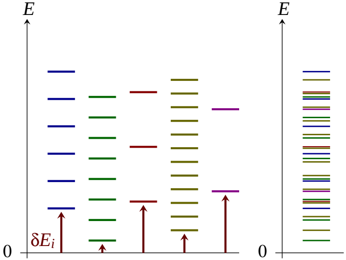

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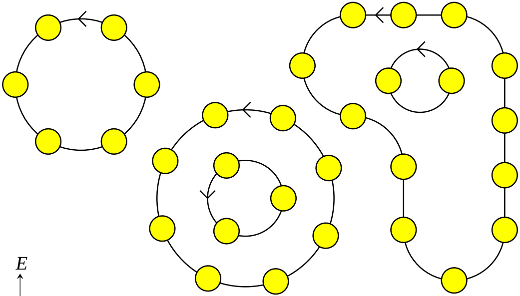
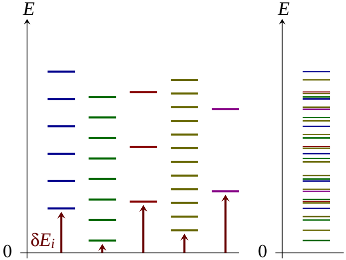
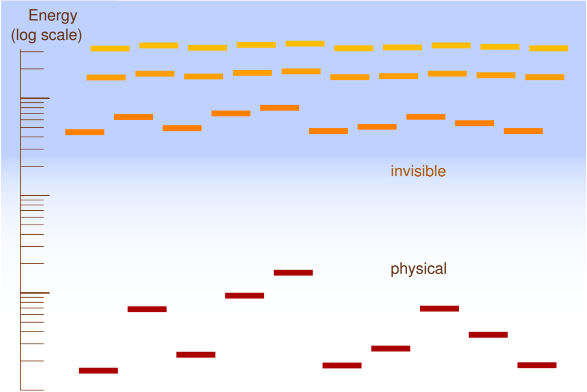




Recently proved: *We can reproduce any quantum system using such models*



Fast fluctuating variables generate sequences of high energy modes. We then expect that only the lowest modes are expected. One then finds that fast fluctuations also arise in the low energy modes. And these accurately reproduce quantum mechanics

But not always the highest energy modes stay empty. What about the Big Bang? What about the inside or the horizon of a black hole?
Why is the dominant part of the universe so close to the *lower bound* of the energy? *Never* the upper bound?
Shouldn't there be a symmetry $E \leftrightarrow E^{\max} - E$?

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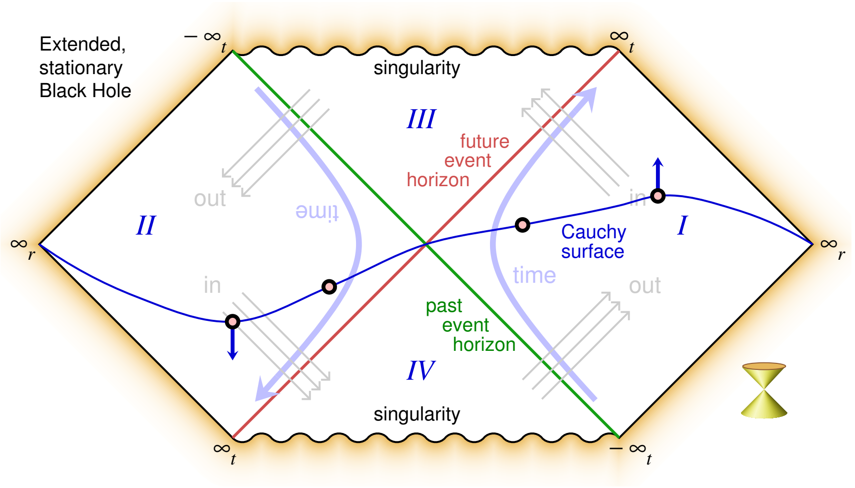
This symmetry is needed in gravity ! At the black hole horizon.



Penrose diagram for eternal black hole (the only Penrose diagram to be used to describe energy eigenstates).

Region *II* describes the *T*- reflection of the black hole. Nature is only invariant under *PCT*. So the is a *PCT* reflection.

At the origin of the Penrose diagram (an $S2$ sphere) spacetime is glued onto its PCT reflection. This turns $S2$ into a *projective sphere.* But, we emphasize that, due to time reflection,

$$
\begin{aligned}
t &\leftrightarrow -t \\
i &\leftrightarrow -i \\
E &\leftrightarrow -E \qquad [?] \\
\text{vacuum state} &\leftrightarrow \text{anti-vacuum} \equiv \text{full state.}
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And *that's* where the imploding matter went: beyond the horizon, space-time is full! Energy did not go to minus energy, but to $E_{\rm max} - E$.

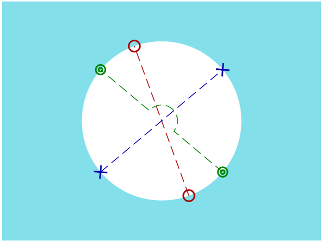
But does $E_{\rm max}$ not generate 'strong' gravitational fields ??
Sure it does !! It modifies the topological shape of space and time by generating the antipodal identification.

Hawking radiation acts similarly at the final stages of a black hole (time reversal symmetry)

Opening up (collapse) and closing in (final evaporation) of a black hole:

Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.



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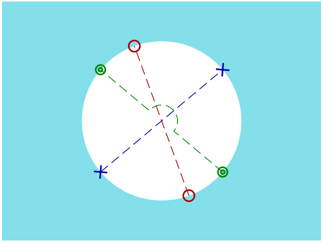

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At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time: *an* **instanton**

N.Gaddam, O.Papadoulaki, P.Betzios (Utrecht former PhD students)

Space coordinates change sign at the identified points

– *and also time changes sign*

(Note: time stands still at the horizon itself).

THANK YOU

Utrecht University

EMMEΦ

Determinism in General Physics and the Foundation of Quantum Mechanics

arxiv: 2010.02019 (5 oct. 2020)

Open Seminar on Theoretical Physics
Moscow Institute of Physics and Technology

Gerard 't Hooft

12 November, 2020

At large time and distance scales the laws of nature appear to be entirely deterministic.

But at the atomic scale, indeterminism seems to emerge:

quantum mechanics.

Whence this mysterious fact? Why are we unable to follow atoms and molecules more precisely when they evolve?

Copenhagen: *do not ask that question, just follow the rules and you get the best predictions that are possible.*

Alas, the predictions come in the form of probabilities.
Like the predictions of the weather.

As in the case of the weather, we can search for microscopic laws that can explain the erratic behaviour, even if we will never do better than the statistical predictions.

We wish to explain where the statistical fluctuations come from.
Is there an underlying, deterministic set of laws? How can we find them?

According to J.S. Bell, CHSH, A.Aspect and many others: *NO !!*

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But I think something is happening that they did not foresee,
and it explains where the stochastic behaviour may come from!

Consider this clue:

an unstable particle, regardless whether it decays in nanoseconds of with lifetimes of billions of years, follows an *exponential decay law.* Can this be squared with determinism?

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Yes ! Just assume that our vacuum is filled with white noise. In practice, this white noise will be completely stochastic, yet we may well assume some deterministic random noise generating agent is responsible, such as: *vacuum fluctuations.*

They do not have to be purely random, any local randomising but deterministic e.o.m. can generate such noise.

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Apparently, we need a theory where the vacuum is a busy place:

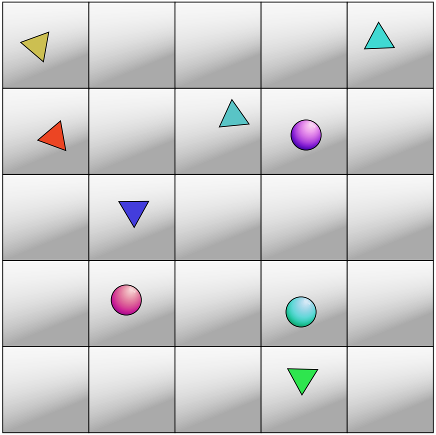
A particle decays when noise from the surrounding "vacuum" makes it decay, either rarely, or quickly.

Can one construct models along such lines?

Yes! and much more. I'll show you how.

Easy to understand – but perhaps not convinving . . . :

The Cellular Automaton: Only *classical* evolution equations.



(Quantum field lattice: same with *quantum* evolution equations)

Claim:

- Every cellular automaton is mathematically equivalent to a genuine quantum field theory on a lattice.
- Every lattice quantum field theory can be accurately approximated by a *classical* cellular automaton.

One needs to understand that every *classical* system can be described in the *quantum language* (Copenhagen) as if it were a *quantum system where the wave function does not spread in time*:

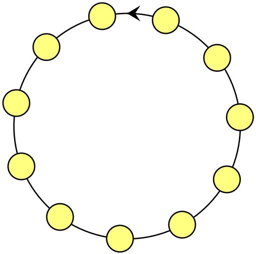
Hamiltonian linear in the momenta.

Operators are arranged in the following classes:

- Beables,
  refer to things that are 'truly there'.
  All beable operators commute with one another, at all times.
- Changeables,
  transform beables into other beables,
- Superimposables,
  all other operators.

But beware, this is only the first step.

Basic Ingredient for Models



using Discrete Fourier transformation:

1. The periodic chain.

Ontological states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

Evolution law:

$$|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$$

$$U(\delta t)|k\rangle = |k+1 \mod N\rangle$$

$$U(\delta t) = e^{-iH\,\delta t}\ , \quad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

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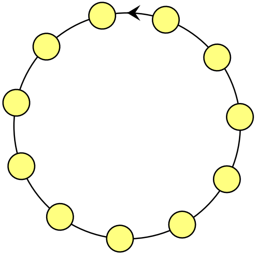

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$$|n\rangle^E \stackrel{\mathrm{def}}{=} \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{2\pi ikn/N}|k\rangle^{\mathrm{ont}}\ , \qquad k=0,\cdots,\ N-1\ ; \quad n=0,\cdots,\ N-1\ .$$

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$$H\ =$$

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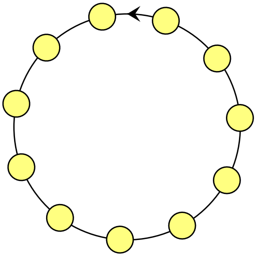

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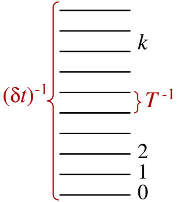
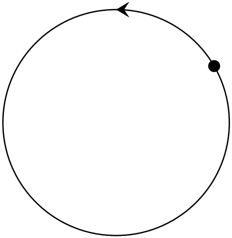
$$H = \frac{2\pi}{N\,\delta t}\,n = \omega n$$





2. The continuum limit.

Ontological states: $|\phi\rangle$

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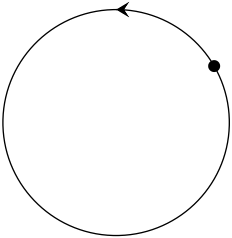

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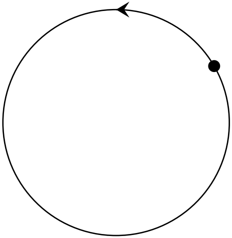
We generate exactly the spectrum of the harmonic oscillator:



2. The continuum limit.

Ontological states: $|\phi\rangle$

Evolution law:

$\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle_t = \omega$

$$U(\delta t)|\phi\rangle = |\phi + \omega t\rangle$$

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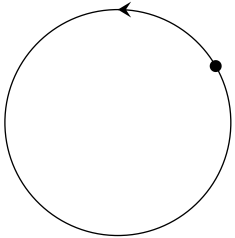

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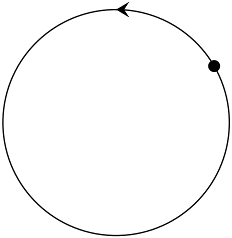

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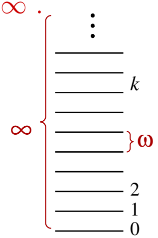

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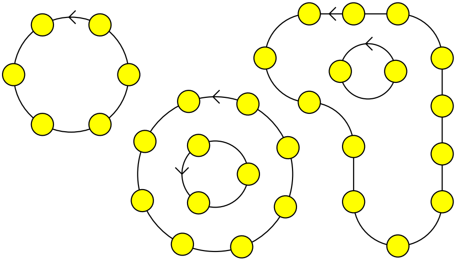
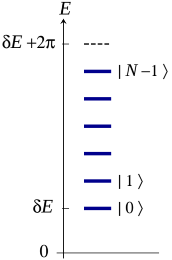
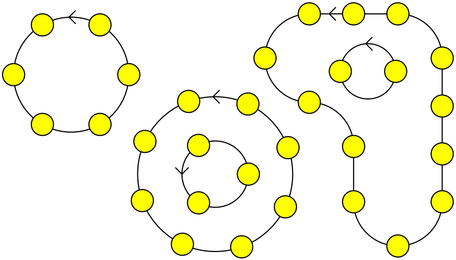


Finite, deterministic, time reversible models





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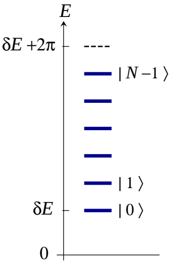

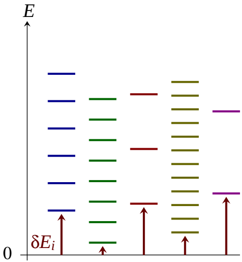

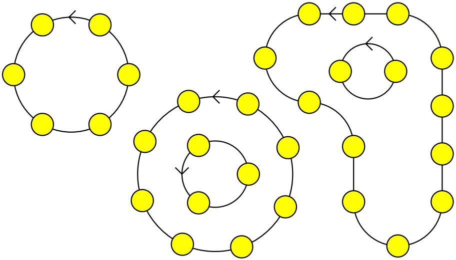

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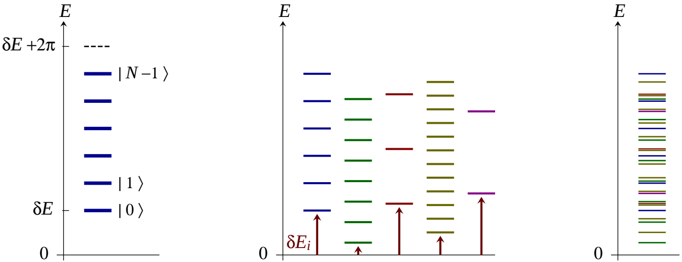

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possibly important in black hole physics,
where the time coordinate flips across the horizon.

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No !

In a theory with *fast fluctuating variables* (the white noise background), these variables, when followed variable by variable, would represent gigantic amounts of energy. But if we assume them to be 'white noise' then we can regard them, all taken together, to be in a zero energy (or very-low-energy) state. This energy cannot be *exactly* observed, but approximately. Thermodynamics will generate equipartition, that is, all states with very low energy per variable, will be strongly preferred.

But then, all other states of the system will become inaccurately defined. We don't know what state the white noise is in, so we don't know which state all other variables are in. We enter into the situation that is only too familiar in QM ...

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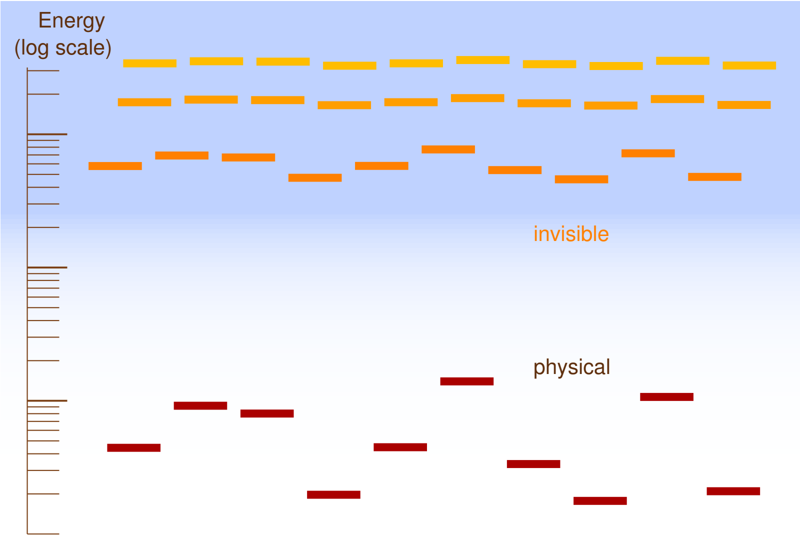
Consider things this way:
The Hamiltonian acts exactly according to the rules of QM. Therefore, there is nothing wrong with *projecting out the lowest N energy states.* Since the Hamiltonian commutes with itself, this does not affect the equations but only our abilities in observing the states, as if we perform time smearing. The states included in our theory represent the heaviest particles observed in LHC but nothing more!


Energy
(log scale)
invisible
physical

The situation that we need to describe occurs when there is a slight inaccuracy in the definition of *time*. Due to energy-time uncertainty,

$$\Delta t \cdot \Delta E \approx \hbar \ ,$$

we find that a slight smearing of time implies that we (have to) ignore the highest energy states.

We do this all the time in our experiments with high energy elementary particles :

We can only observe the low energy particles. To unravel the high energy, or very massive elementary particles, we need accelerators with energies that we cannot reach.

We cannot follow the fast variables!

Thus, curiously, in the classical limit – read: large scale limit – energy *does* become an observable. But it does not commute with our original beables.

This creates a new – and interesting – situation, which can indeed occur in ordinary classical theories.

I describe what happens in more detail in:

See arxiv:2010.02019 .

There, I construct a completely classical model, which behaves *exactly* as a quantum system.

Here, I present an outline.

Consider a quantum system with a finite dimensional ‘Hilbert’ space of states. The Hamiltonian is an arbitrary, $N \times N$ hermitean matrix. We construct a model that will generate this matrix as an ‘effective’ or ‘emergent’ quantum Hamiltonian.

We assume N classical, fast variables, one for every state of the system.

This sounds like a lot, but we are thinking of the vacuum fluctuations of a high-mass elementary particle. It has independent field degrees of freedom in every small volume element of space.
This suffices, and we can economise later (multiple use of a given fast variable)

Each fast variable i lives on a circle with period L_i.

Take L_i discrete, like in our elementary unit model ($\rightarrow$ N-dimensional torus).

Take the different L_i to be relative primes.

All periods L_i are much shorter than the inverse mass of the slow objects (particles).

e.o.m.: $x_i(t+1) = x_i(t) + 1 \mod L_i$.

This is driven by the Hamiltonian:

$$H = \sum_i p_i \ , \quad p_i = \frac{\partial}{\partial x_i} = \frac{2\pi n_i}{L_i} \ , \qquad n_i = 0, 1, \cdots, L_i - 1 \ .$$

Assume an *even distribution* of these variables. This means that, in our *formal* quantum language, they are all in their ground states. To make the distribution not even, we need the excited states, but their energies, are at least $2\pi/L_i$, which we take to be much larger than the energies of our quantum states.

The quantum degrees of freedom that I want to describe next, consist of N classical states.

We start with having no evolution at all there, so, for the classical states,

$$H_{\rm class} = 0.$$

Now consider two states, i and j. Assume that I want to add δH_{ij} to my Hamiltonian. There are three possible forms:

$$H = \alpha_1\sigma_1 + \alpha_2\sigma_2 + \alpha_3\sigma_3,$$

$$\text{with} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

If we impose $\alpha_1 = \frac{1}{2}\pi$, $\alpha_2 = \alpha_3 = 0$, then this can easily be seen to be a classical evolution: